Motion of a sphere in an oscillatory boundary layer: an optical tweezer based study

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Abstract

The drag forces acting on a single polystyrene sphere in the vicinity of an oscillating glass plate have been measured using an optical tweezer. The phase of the sphere is found to be a sensitive probe of the dynamics of the sphere. The evolution of the phase from an inertially-coupled regime to a purely velocity-coupled regime is explored. Moreover, the frequency dependent response is found to be characteristic of a damped oscillator with an effective inertia which is several orders of magnitude greater than that of the particle.

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INTRODUCTION

Study of the forces between two sliding bodies in the presence of an intervening fluid is an important problem in fluid mechanics [1]. In recent years, a variety of experimental and theoretical/ computational tools have been brought to bear of this central issue. Experimental studies of the "simple" case of hydrodynamic drag forces acting on a single particle suspended in a liquid near a plate executing sinusoidal oscillations are rare, although there have been measurements on the lift forces acting on spheres in oscillatory flows [3]. Such measurements have shown a highly non-monochromatic response of the lift force as a function of the oscillating frequency. This problem is important in understanding the role played by colloidal particles in a lubricating liquid as well as the nature of interactions between the colloidal particles and the solid interface formed say by the walls of narrow channels, e.g., blood vessels.

In this paper we present a new technique based on an optical tweezer that addresses an important aspect of this problem: how does one probe the dynamical response of a solid particle in "contact" with a substrate in the presence of a lubricating liquid, which requires an elucidation of the notion of the "contact" itself. The study reveals the importance of the phase response of the particle which provides insight into the nature of the dynamics.

A schematic of the experimental setup is shown in figure 1 . A very dilute monodisperse colloidal solution of polystyrene spheres of radius(a) of 1μm (Alfa Aesar) was placed between two parallel glass plates separated by a distance of 2mm. The top glass plate was kept stationary. The bottom glass plate was attached to a xyz piezo stage subjected to an oscillatory motion in the y direction with an amplitude y_{po} and frequency ω, i.e., $y_p = y_{po}\sin(\omega t)$, where y_p is the instantaneous displacement of the glass plate . A colloid particle was trapped near the glass plate using an IR laser of wavelength 1064 nm focused using a 100X oil immersion objective lens. A He-Ne laser of wavelength 632.8 nm was used to track the position of the sphere. The position of the sphere was detected using a quadrant photo diode (UDT instruments). The positional response of the sphere was then locked on to the drive signal of the piezo plate using a SRS 830 lock-in amplifier and both phase and amplitude of the locked in signal were analyzed. The separation between the particle and the oscillating glass plate was varied by moving the piezo stage in the z direction. The trap stiffness and the corner frequency of the trap were analyzed using the power spectrum of

the equilibrium positional fluctuations of the trapped particle.

FORCES ACTING ON THE SPHERE DUE TO OSCILLATORY FLOW

The two extreme scenarios possible in the problem are (i) when the sphere is in physical contact with the bottom plate and (ii) when the sphere is far away from the glass plate. In the first scenario the coupling between the sphere and the bottom plate is mainly inertial and this is reflected in an "in-phase" response of the sphere with respect to the motion of the bottom plate. When the sphere is far away from the plate then the forces acting on the sphere will be dominantly due to viscosity mediated shear stresses and drag forces arising from the motion of the liquid. Therefore, in this experiment we have two control parameters, i.e., the height of the sphere from the plate and the velocity of the plate.

The motion of the liquid in presence of oscillatory shear is governed by the Naiver Stokes equation $\rho\left(\frac{\partial v}{\partial t}+(v.\nabla)v\right) = -\nabla p+\eta\nabla^2 v$, where v is the velocity, η is viscosity ,∇p is the pressure gradient and ρ is the density of the liquid. For the present problem there is no pressure gradient, i.e., $\nabla p = 0$, and equation of continuity demands $\nabla\cdot\mathbf{v} = 0$, thus $(v.\nabla)v = 0$. With the above assumptions the Naiver Stokes equation can be written as $\rho\frac{\partial v_y}{\partial t} = \eta\frac{\partial^2 v_y}{\partial z^2}$. This has traveling wave solutions of the form, $v_y(z,t) = v_0 e^{i(kz-\omega t)}$ with Stokes boundary layer, $\delta = \sqrt{\frac{2\eta}{\omega\rho}}$ and $k = \pm\frac{(1+\imath)}{\delta}$ [5] . The frictional force acting on the sphere due to the velocity gradient is in the direction of the liquid motion. The force per unit area is given by

$$S = \eta\frac{\partial v}{\partial z} = -\frac{\sqrt{2}}{\delta}\eta v_0 e^{-i\left(\omega t+\frac{\pi}{4}\right)}e^{(i-1)\frac{z}{\delta}}. \tag{1}$$

We obtain the total frictional force (F_s) acting on the sphere by computing the surface integral.

$$F_s = 2\pi\eta a v_0 e^{-\left(\frac{h+a}{\delta}\right)}(1-e^{\frac{2a}{\delta}})e^{-i\omega t}, \tag{2}$$

where h is the height of the sphere from the glass plate.

The forces experienced by the trapped particle at a height h from the glass plate are (i) the spring like force $-k_{op}x$, exerted by the tweezer; (ii) the viscous drag force $-6\pi\eta a(\dot{x}-u)$,where $\dot{x}$ is the velocity of the sphere and u the velocity of the liquid in the vicinity of the sphere.(ii)hydrodynamic force F_h and (iv) a frictional force (F_{plate}) due to the interaction with the plate.

Thus, the equation of motion of the sphere is

$$m_{eff}\ddot{x} + 6\pi\eta a\dot{x} + k_{op}x = F_h + F_{plate} + 6\pi\eta a u, \tag{3}$$

where m_{eff} is the effective mass of the particle. We introduce m_{eff} in place of actual mass of the particle to account for the effects of frequency, hydrodynamic interactions, and presence of wall to the Stoke's drag. The force acting on the sphere due to the glass plate is mainly electrostatic in origin and hence strongly dependent on the sphere-plate separation. When the sphere is far away from the plate and frequency of oscillations are low, F_h is the frictional shear force F_s. However, in cases where the sphere is within the stokes boundary layer and the length scale of the vortex shedding is greater than the plate-sphere separation one expects flow field around the particle to be altered by the presence of bottom plate and the inertial effects to differ from those for a particle in an unbounded flow [2, 4].Thus, for the sphere near the bottom plate the hydrodynamic force will differ from the simple minded frictional shear force.

EXPERIMENTAL RESULTS

We will first discuss the results which show transition of the motion of the sphere dominated by inertial contact with the plate to the regime where the motion of the sphere is mainly driven by the liquid velocity. Top panel of Fig. 2 shows the motion of the sphere (solid line) suspended in water and the corresponding drive signal to the y-direction of the piezo (dotted line). The data plotted is for a constant driving frequency (1Hz) and constant drive amplitude($1\mu m$) for various sphere-plate separation (h) [6]. The drive signal shown in the top panel is only a guide to the eye and the value on the y axis is not a reflection of its absolute magnitude. When h (~ 0.1 μ m) is small, the response of the sphere is in phase with the drive signal. As the plate-sphere separation is increased the motion of the sphere develops a phase lag with respect to the plate. This is shown in the bottom panel of Fig. 2 where the histogram of the phase of the motion of the sphere(averaged over 3 seconds) is plotted as a function of the sphere-plate separation.“ Phase ” of the motion of the sphere is defined by the phase of the locked in signal with respect to the driving signal. The phase of the sphere's motion confirms the two regimes-inertia coupled (in-phase) and velocity coupled (finite phase lag). In between the two extremes of in phase and $\sim \pi/2$ out of phase motion,

the histogram of the phase of the motion of the sphere shows a broad distribution [7].

We now present our results which measures the effect of large drive frequencies on the motion of the sphere for appreciably large values of h. In this case the sphere executes a $\pi/2$ out of phase motion with respect to the motion of the bottom plate for low drive frequencies. This confirms that the motion of the sphere is predominantly velocity coupled. We have used two aqueous mediums of suspension, namely water($\eta = 1mPas$) and glycerol($\eta = 760mPas$) for this study. Left panel of Fig. 3 shows response of the sphere(solid spheres joined by line) suspended in glycerol as the frequency of the oscillation of the plate is varied when the sphere-plate separation is constant at about ~ 1.3 μm (sphere is far away from the plate).The drive amplitude of the plate was kept constant at $0.1\mu m$. The trace of the velocity of the liquid is shown by a solid line. Note that the phase of the velocity of the liquid is $\pi/2$ phase shifted with respect to that of the bottom plate.[8] It can be seen that at 2Hz the sphere moves almost in phase with the velocity of the liquid. However, as the drive frequency is increased the sphere develops a $\sim \pi/2$ phase difference for drive frequency of 11Hz and $\sim \pi$ phase difference for a drive frequency of 40 Hz. The corresponding average phase is shown by solid circles joined by line in top panel of Fig.4. The inset of the top panel in fig. 4 shows the amplitude of the locked in signal.

The right panel of Fig. 3 shows response of the sphere(solid spheres joined by line) suspended in water as the frequency of the oscillation of the plate is varied when the sphere-plate separation is constant at about $\sim$ 1.3 μm (sphere is far away from the plate).The drive amplitude of the plate was kept constant at $1\mu m$. The sphere's motion at the driving frequency of 2Hz is purely sinusoidal and monochromatic, with a dominant frequency of 2Hz. At this frequency the motion of the sphere is almost in phase with the velocity of the liquid. However as the frequency is increased, the motion of the sphere develops a phase lag with respect to the velocity of the liquid. This is also accompained by a large distortion of the waveform of the motion of the sphere. It is interesting to note that such distortions are absent for the case of glycerol.The corresponding average phase is shown by solid circles joined by line in bottom panel of Fig.4. The inset of the bottom panel in fig. 4 shows the amplitude of the locked in signal.

In the analysis of the phase as shown in fig.4 we have shifted the phase of the drive signal by $\pi/2$ to account for the velocity contribution. That is, the phase is plotted with respect to the velocity of the liquid. As the frequency is increased from 2Hz to 60Hz the phase decreases

monotonically from zero to $-\pi$. Since the sphere is far away from the plate we ignore F_{plate}. We can then consider this system as a forced damped harmonic oscillator. The equation of motion of a forced damped harmonic oscillator is $\ddot{x}+\gamma\dot{x}+\omega_0^2 x=\frac{F_0}{m}\cos\omega t$ which has a steady state solution $x=A\cos\omega t$ where $A=\frac{F_0}{m}\frac{1}{\left[(\omega_0^2-\omega^2)^2+(\omega\gamma)^2\right]^{1/2}}$ and $\phi=\arctan\left(\frac{\gamma\omega}{\omega^2-\omega_0^2}\right)$. Comparing it with eq no 3 we see that in our case, $\gamma=\frac{6\pi\eta a}{m_{eff}},\omega_0^2=\frac{k_{op}}{m_{eff}}$.

Figure 4 also shows the fit (open squares joined by line) to Eqn., $\phi=\arctan\left(\frac{\gamma\omega}{\omega^2-\omega_0^2}\right)$ to the experimental data. From the fit we obtain $\gamma=150$ and $\omega_0=11Hz$ for glycerol and $\gamma=220$ and $\omega_0=20Hz$ for water. This implies implies $m_{eff}=0.095\text{x}10^{-6}$Kg for the case of glycerol and $m_{eff}=0.085\text{x}10^{-9}$Kg for water. The value of $k_{op}=m_{eff}\omega_0^2$ obtained from the fit parameters for glycerol is $5.96\text{x}10^{-4}$N/m which agrees well with that obtained from the equilibrium positional fluctuations of the sphere($2.7\text{x}10^{-4}$N/m).The value of $k_{op}=m_{eff}\omega_0^2$ obtained from the fit parameters for water is $13.4\text{x}10^{-7}$N/m which agrees well with that obtained from the equilibrium positional fluctuations of the sphere($11.4\text{x}10^{-7}$N/m). The quality of the fit improves if m_{eff} is considered as function of effective mass. But in that case the equation of motion should be solved using perturbation theory.

Notice that m_{eff} is much larger than the true mass of the bead($\sim 4.18\text{x}10^{-15}$Kg). We can understand this by saying that it is not just the sphere but also the fluid around it which is behaving like an oscillator.

DISCUSSION

The relevant length scales perpendicular to direction of the flow, that is, in the z-direction are the particle diameter,($2\mu m$), and the stokes boundary layer, $\delta=\sqrt{\frac{2\eta}{\omega\rho}}$. In frequency range,$1Hz\ldots 60Hz$, covered in our experiments $\delta=13mm\ldots 1.6mm$ for glycerol and $\delta=0.5mm\ldots 0.07mm$ for water. In the direction parallel to the flow, one of the relevant length scales is again the particle diameter ($2\mu m$) and the other is associated with vortex formation, shedding and potential interactions. For a sinusoidal motion, the length scale associated with vortex formation is roughly equal to amplitude of the plate motion [2] which in our experiments is about $0.1\mu m$ for glycerol and $1\mu m$ for water.

The motion of the sphere comprises of two kinds of degrees of freedom- translation and rotational. Translation motion arises because whole fluid around it translates when the bottom plate is sinusoidally driven. Since the sphere is trapped and is subjected to shear

stresses there will be a finite torque leading to the rotation of the sphere about the x axis with an angular velocity$\mathbf{\Omega} = (\Omega, 0, 0)$. No-slip boundary condition on the sphere's surface ensures that the curl of the velocity of the liquid is non zero around the surface of the sphere. The Navier Stokes equation which defines the flow of the liquid about the sphere is given by $\nabla^2 \mathbf{q} - \nabla p = \frac{\rho}{\eta} (\mathbf{q}.\nabla) \mathbf{q} + \frac{\rho}{\eta} \frac{\partial \mathbf{q}}{\partial t}$, where $\mathbf{q} = (\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3})$ is the velocity of the fluid. The origin of the cartesian coordinate is taken to coincide with the center of the sphere. The boundary conditions are $\mathbf{q} \rightarrow 0$ when z tends to infinity, $\mathbf{q} = \mathbf{\Omega} \times \mathbf{r}$ for the flow on the sphere and $\mathbf{q} \rightarrow$ velocity of the plate for the flow on it, here $\mathbf{r} = (\mathbf{x}, \mathbf{y}, \mathbf{z})$.This will lead to vortex formation which decays with a length scale of the boundary layer thickness. It is noteworthy that the stokes boundary layer is of the order of at least tens of microns for both glycerol and water in the frequency range covered in our experiments. Since the sphere is within the boundary layer, one expects considerable potential interactions between the rotational flow of the liquid and the oscillating plate. This scenario could possibly result in an inertial coupling between the plate and the sphere. This in turn could increase the effective mass of the sphere and hence exhibit the observed phase behavior.

CONCLUSION

To our knowledge,there has been no experimental/computational fluid dynamics simulations of a particle, rotating as well as translating, in an oscillatory flow. Fischer *et al* have performed calculation of lift and drag forces on a stationary sphere subjected to a pressure driven oscillatory flow. Our results are the first direct measurements of the drag forces acting on a rotating sphere subjected to an oscillatory motion. We find that the phase of the motion of the sphere with respect to the drive is a sensitive tool to study its dynamics. We have been able to explain our data in terms of a damped harmonic oscillator. The effective mass that comes out of the calculations is orders of magnitude greater than the bare mass of the sphere. This highlights the importance of the role of inertia in an otherwise viscosity dominated flow. We believe that optical tweezers will be a effective tool to address fluid dynamics problems at small length scales.

[1] L. D. Landau and E. M. Lifshitz, *Fluid mechanics*, (Second edition1987).

[2] P. F. Fischer, G. K. Leaf, and J. M. Restrepo, *Forces on particles in oscillatory boundary layers*, Journal of Fluid Mechanics **468** (2002), 327–347.

[3] G. N. Rosenthal and J. F. A. Sleath, *Measurements of lift in oscillatory flow*, J. Fluid Mechanics **164** (1986), 449.

[4] B. M. Sumer, B. L. Jensen, and J. Fredse, *Effect of a plane boundary on oscillatory flow around a circular cylinder*, Journal of Fluid Mechanics **225** (1991), 271–300.

[5] We have ignored the reflected travelling waves considering the top plate to be at infinity. This is because the distance between the plates is much larger compared to the boundary layer.

[6] The heights have been calculated from the z displacement of the piezo.

[7] On carefully inspecting the displacement of the sphere one finds the sphere to show slip-stick behavior.

[8] $y_p = y_{p0} sin(\omega t)$, $\dot{y}_p = y_{p0}\omega sin(\omega t + \pi/2)$, where $\dot{y}_p$ is the velocity of the plate and assuming no-slip boundary condition is also the velocity of the liquid.

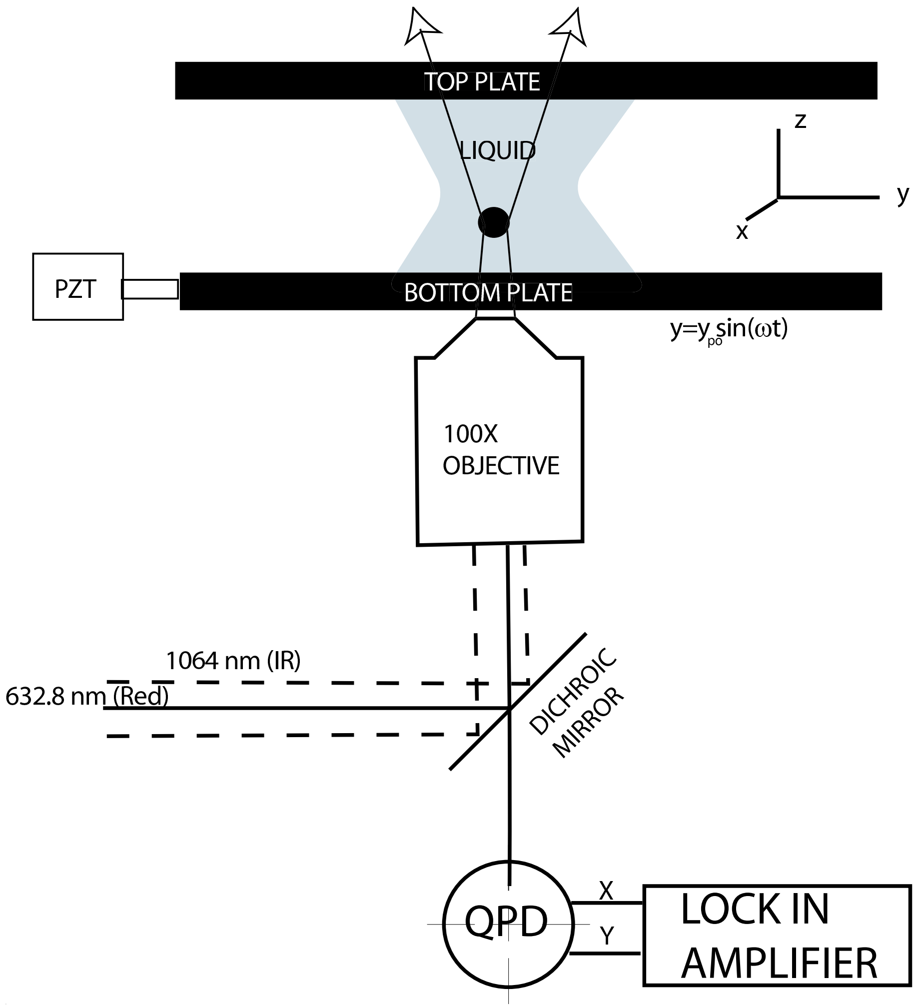

FIG. 1: Schematic of the experimental setup. Liquid containing $2\mu m$ colloidal particles is held between two glass plates separated by 2mm. The particle is optically trapped at a height h from the bottom plate. The lower plate is subjected to an oscillatory motion $y_p = y_{po}\sin(\omega t)$, where y_p is the instantaneous displacement of the glass plate, ω is angular velocity and y_{po} is the amplitude. The top plate is kept fixed.

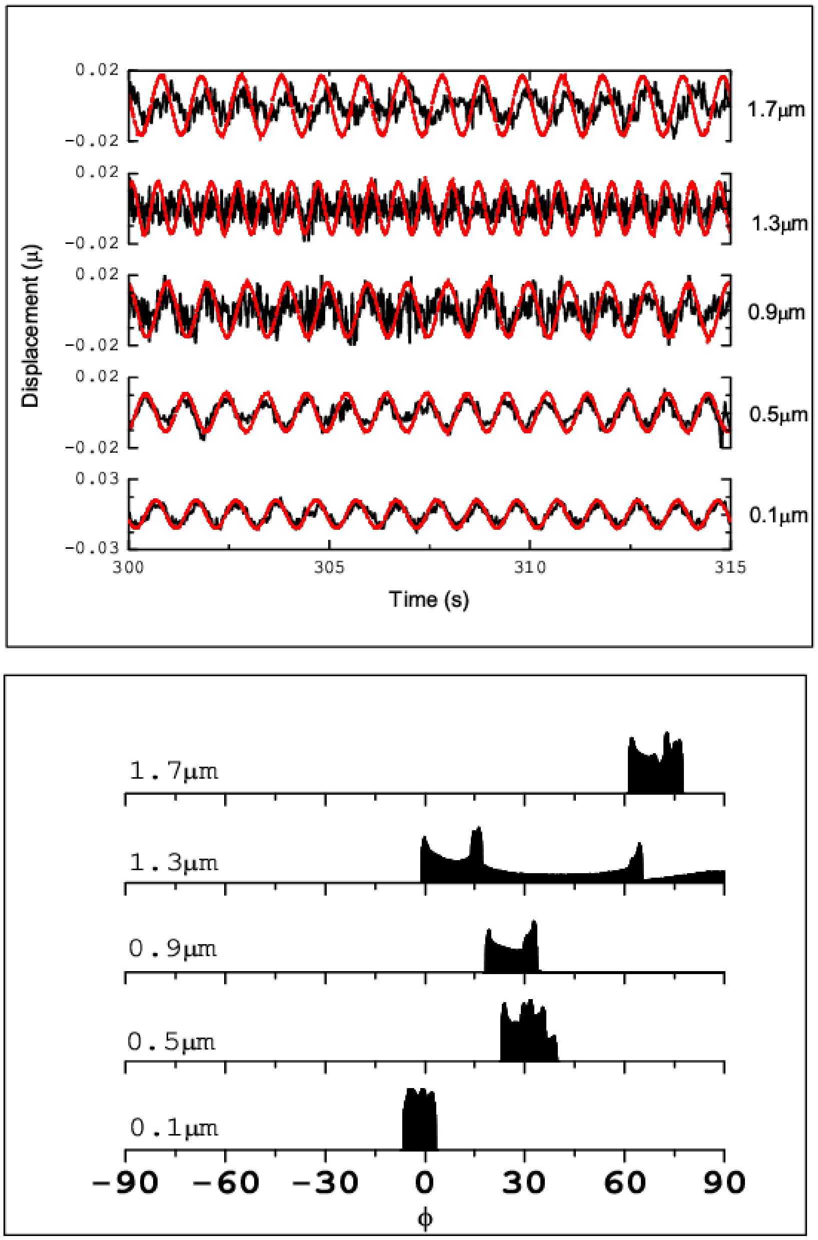

FIG. 2: Top : The motion of the sphere(solid line) as a function of the sphere-plate separation: (a)$\sim$ 0.1 μm (b)$\sim$ 0.5 μm (c)$\sim$ 0.9 μm (d) $\sim$ 1.3 μm. (e)$\sim$1.7 μm at a fixed driving frequency of 1Hz and amplitude of 1μm. The trace showing the motion of the plate (dotted line) is a guide to the eye, and the corresponding y axis is not a reflection of its absolute amplitude. Bottom : The corresponding histograms of the phase of the locked in signal as a function of the sphere-plate separation. The y axis for the histograms is in logarithmic scale.

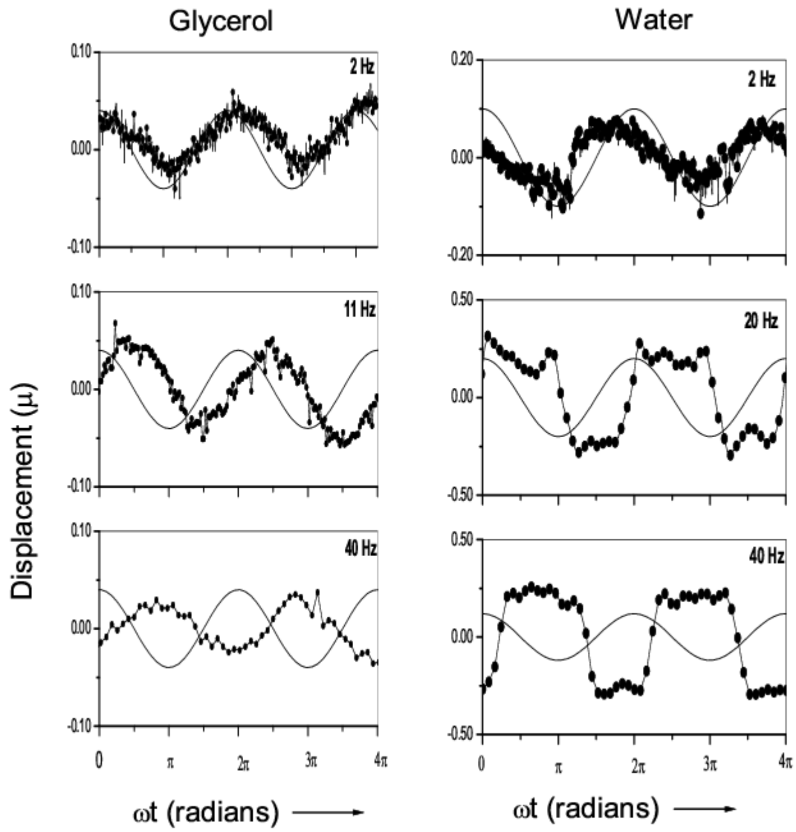

FIG. 3: Displacement of the sphere (solid circles joined by lines) in response to the velocity of the liquid (Solid line) for few select frequencies. The frequencies are shown by the side of each trace. Left: The data shown is for glycerol, Right : The data shown is for water. The trace showing the velocity of the liquid is a guide to the eye, and the corresponding y axis is not a reflection of its absolute amplitude.

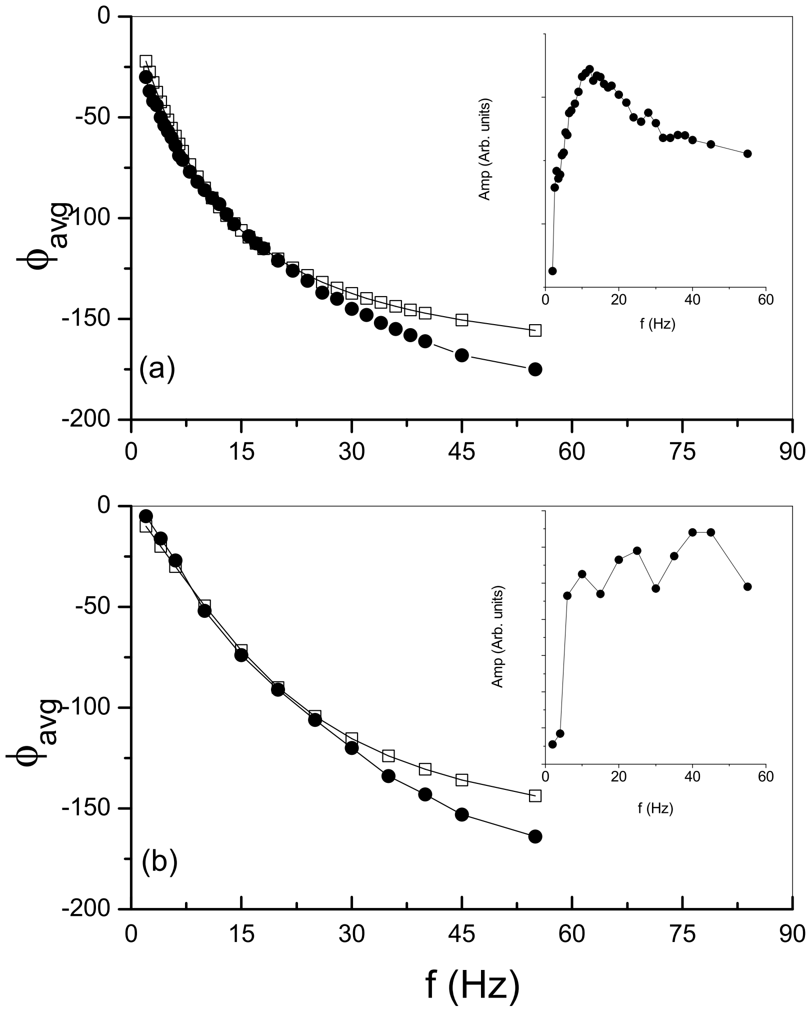

FIG. 4: The variation of average phase with respect to the velocity of the liquid. The inset shows the amplitude of the locked in signal. Top: The data shown is for glycerol. Bottom: the data shown is for water